

Check-Up

13

Using a four-function calculator, precedence of operators

- a) On your calculator enter this calculation just as it is written, reading from left to right: $3 + 4 \quad 5 = \dots$ Does your calculator give you the 'right' answer?
- b) There are 149 pupils from Key Stage 1, 344 pupils from Key Stage 2 and 34 adults, going on a school trip. To calculate how many 68-seater coaches are required, the headteacher enters on a calculator: $149 + 344 + 34 \div 68 = \dots$ Is this correct?

Answers to check-up 13

- a) Your calculator may give the answer 35. Technically this is wrong. The correct answer is 23.
- b) If a scientific calculator is used, the answer obtained (494 coaches!) will be incorrect. If a basic four-function calculator is used, the result displayed is 7.75, which means 8 coaches are required. This is correct.

Discussion and explanation of check-up 13

How can a calculator give you the wrong answer? Well, it depends on what kind of a calculator you are using. The problem is that $3 + 4 \times 5$ is ambiguous. Does it mean 3 added to (4×5) , or $(3 + 4)$ multiplied by 5? In formal mathematics there is a convention that, unless there are brackets to indicate otherwise, division and multiplication have 'precedence' over addition and subtraction. So, being pedantic, we would have to say that $3 + 4 \times 5$ should mean that you multiply the 4 by the 5 first and then add 3 to this, i.e. $3 + (4 \times 5)$. This convention is especially necessary when manipulating and using algebraic expressions. Normally, in arithmetic calculations the context makes it clear which bits of a mixed calculation should be done first. If there is any possibility of ambiguity it makes sense to put in whatever brackets are necessary. If you mean to do the $3 + 4$ first, write $(3 + 4) \times 5$. If you mean to do the multiplication first, write $3 + (4 \times 5)$.

Now, if your calculator gives you the answer 23 when you enter $3 + 4 \times 5 =$, then you have a scientific calculator that uses what is called an algebraic operating system. In this system, when you ask the calculator to do an addition (like $3 + 4 \dots$) it waits to see if there is a multiplication or division coming next before proceeding. If there is, it does that first! If, however, you get the answer 35, then you have a basic four-function calculator that ignores the mathematician's convention about precedence of operators and just does the operations in the order they are entered. On-screen calculators on personal computers are likely to be of the basic four-function variety. Note also that many of these use an asterisk (*) and a forward slash (/) as symbols for multiplication and division respectively.

So, if the headteacher in (b) enters ' $149 + 344 + 34 \div 68 =$ ' onto a four-function calculator, it will (correctly in this context) add up the first three numbers

and then divide by 68. But a scientific calculator would calculate ' $34 \div 68$ ' first before adding it to the sum of 149 and 344. To get a scientific calculator to do the calculation required here, we would have to press the equals key after the 34, to get the addition completed before we do the division by 68.

Summary of key ideas

- ◆ There is a convention in formal mathematics that, unless otherwise indicated by brackets, divisions and multiplications in mixed calculations have precedence over additions and subtractions.
- ◆ In writing down a mixed calculation it is best to use brackets to avoid ambiguity about which parts of the calculation should be done first.
- ◆ A scientific calculator uses the precedence-of-operators convention.
- ◆ A basic four-function calculator just does the operations in the order in which they are entered.

Further practice

- 13.1** What would be displayed if you entered ' $12 - 6 \div 2 =$ ' on (a) a basic four-function calculator, (b) a scientific calculator with an algebraic operating system?
- 13.2** For the school trip in Check-up 13 question (b), the headteacher wants to check the child–adult ratio and enters on a calculator: $149 + 344 \div 34 =$. This gives the result 14.5. What kind of calculator is being used?